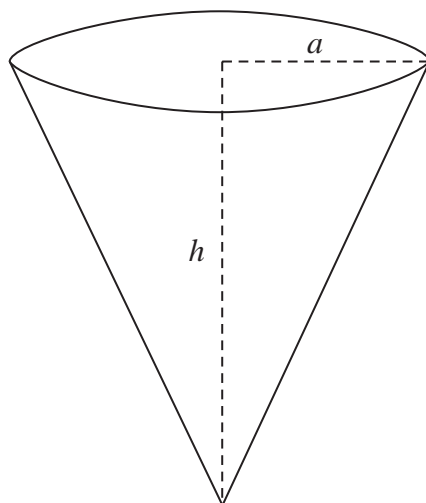


1.

**Figure 1**

A hollow right circular cone, of base radius a and height h , is fixed with its axis vertical and vertex downwards, as shown in Figure 1. A particle moves with constant speed v in a horizontal circle of radius $\frac{1}{3}a$ on the smooth inner surface of the cone.

Show that $v = \sqrt{\left(\frac{1}{3}hg\right)}$. (7)



2. A particle of mass 4 kg is moving along the horizontal x -axis under the action of a single force which acts in the positive x -direction. At time t seconds the force has magnitude

$$\left(1 + 3t^{\frac{1}{2}}\right) \text{ N.}$$

When $t = 0$ the particle has speed 2 m s^{-1} in the positive x -direction. Find the work done by the force in the interval $0 \leq t \leq 4$

(7)



3. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N . The other end of the spring is attached to a fixed point A . The particle P is held at rest at the point B , which is 1 m vertically below A , and then released.

(a) Find the acceleration of P immediately after it is released from rest.

(4)

The particle comes to instantaneous rest for the first time at the point C .

(b) Find the distance BC .

(6)



4. A particle P is moving along the positive x -axis. At time t seconds, $t \geq 0$, P is x metres from the origin O and is moving away from O with velocity $v \text{ m s}^{-1}$, where $v = \frac{4}{(x + 2)}$. When $t = 0$, P is at O . Find

(a) the distance of P from O when $t = 2$ (5)

(b) the magnitude and direction of the acceleration of P when $t = 2$ (5)



5.

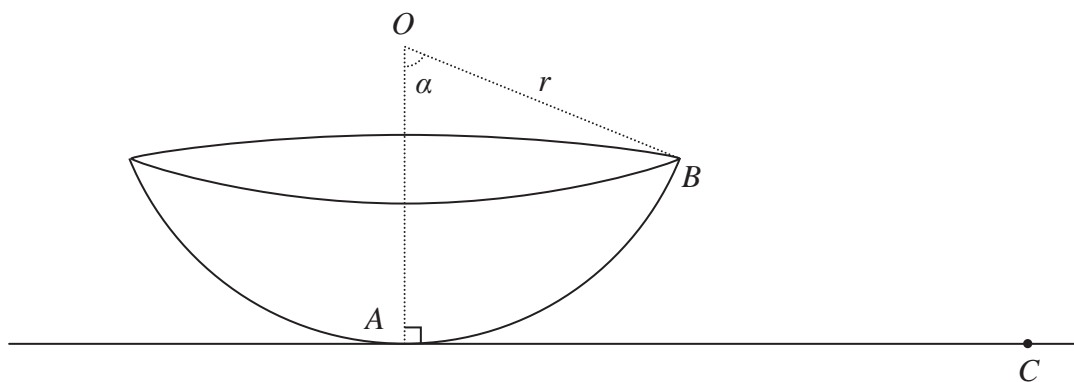


Figure 2

Part of a hollow spherical shell, centre O and radius r , forms a bowl with a plane circular rim. The bowl is fixed to a horizontal surface at A with the rim uppermost and horizontal.

The point A is the lowest point of the bowl. The point B , where $\angle AOB = \alpha$ and $\tan \alpha = \frac{3}{4}$, is on the rim of the bowl, as shown in Figure 2. A small smooth marble M is placed inside the bowl at A , and given an initial horizontal speed \sqrt{gr} . The motion of M takes place in the vertical plane OAB .

(a) Show that the speed of M as it reaches B is $\sqrt{\left(\frac{3}{5}gr\right)}$. **(4)**

After leaving the surface of the bowl at B , M moves freely under gravity and first strikes the horizontal surface at the point C . Given that $r = 0.4$ m,

(b) find the distance AC . **(8)**



6. (a) A uniform lamina is in the shape of a quadrant of a circle of radius a . Show, by integration, that the centre of mass of the lamina is at a distance of $\frac{4a}{3\pi}$ from each of its straight edges. (7)

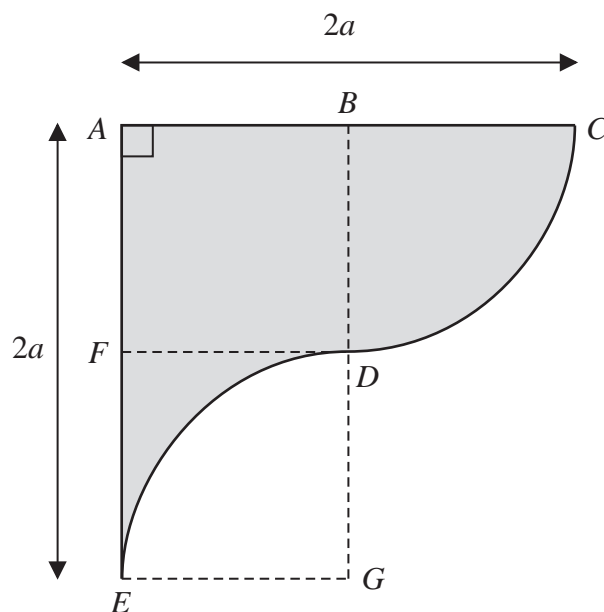


Figure 3

A second uniform lamina $ABCDEFA$ is shown shaded in Figure 3. The straight sides AC and AE are perpendicular and $AC = AE = 2a$. In the figure, the midpoint of AC is B , the midpoint of AE is F , and $ABDF$ and $DGEF$ are squares of side a . BCD is a quadrant of a circle with centre B . DGE is a quadrant of a circle with centre G .

- (b) Find the distance of the centre of mass of the lamina from the side AE . (5)

The lamina is smoothly hinged to a horizontal axis which passes through E and is perpendicular to the plane of the lamina. The lamina has weight W newtons. The lamina is held in equilibrium in a vertical plane, with A vertically above E , by a horizontal force of magnitude X newtons applied at C .

- (c) Find X in terms of W . (3)



7. Two points *A* and *B* are 4 m apart on a smooth horizontal surface. A light elastic string, of natural length 0.8 m and modulus of elasticity 15 N, has one end attached to the point *A*. A light elastic string, of natural length 0.8 m and modulus of elasticity 10 N, has one end attached to the point *B*. A particle *P* of mass 0.2 kg is attached to the free end of each string. The particle rests in equilibrium on the surface at the point *C* on the straight line between *A* and *B*.

(a) Show that the length of *AC* is 1.76 m. (4)

The particle *P* is now held at the point *D* on the line *AB* such that $AD = 2.16$ m. The particle is then released from rest and in the subsequent motion both strings remain taut.

(b) Show that *P* moves with simple harmonic motion. (4)

(c) Find the speed of *P* as it passes through the point *C*. (2)

(d) Find the time from the instant when *P* is released from *D* until the instant when *P* is first moving with speed 2 m s⁻¹. (4)



